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Plane-stress fracture of polycarbonate films

S. HASHEMI
The University of North London, London School of Polymer Technology, Holloway Road, London N7 8DB, UK

The use of the specific essential work of fracture, w_e, to characterize fracture of polycarbonate films is described. It is shown that the plane-stress specific essential work of fracture for polycarbonate film can be obtained from single-edge-notched-tension specimens, by extrapolating the straight-line relationship between the total work of fracture, W_f, and ligament length, L, to zero ligament length. From the data, it seems that, for a given film thickness, w_e is almost independent of the specimen width but increases with increasing thickness. The non-essential work of fracture as obtained from the slope of a W_f versus L plot showed no significant width dependence, and for the majority of thicknesses it was almost invariant with thickness, indicating that the shape of the outer plastic zone surrounding the fracture process zone is almost invariant with the dimensions of the test specimen.

1. Introduction

The thickness dependence of toughness is related to the gradual transition from the full-plane-strain to the full-plane-stress state. When the surface region where the plane stress prevails become small in thick sections, its influence can be neglected and the behaviour becomes independent of thickness. In thin sections the plane-stress region is not small in comparison to the plane-strain region, and the nominal stress at fracture increases with the increasing ratio between the size of the plane-stress and plane-strain regions. The dependence of the fracture toughness, K_c, upon thickness is given diagrammatically in Fig. 1. Beyond a certain thickness, B_c, a state of plane strain prevails and the toughness reaches the plane-strain value, K_{1c}, which is independent of the thickness. According to [1] the thickness, B_c, may be determined from the following criterion:

$$B_c = 2.5\left(\frac{K_{1c}}{\sigma_y}\right)^2 \quad (1)$$

where σ_y is the tensile yield stress of the material. As shown in Fig. 1, there is an optimum thickness, B_0, where the toughness reaches its highest level. This level is usually considered to be the real plane-stress fracture toughness. B_0 may be estimated from [2]

$$B_0 = \frac{K_{1c}^2}{3\pi\sigma_y^2} \quad (2)$$

In the transitional region between B_0 and B_c, the toughness has intermediate values. For thicknesses below B_0 there is uncertainty about the toughness. In some cases a horizontal level is found [3], in other cases a decreasing K_c-value is observed [4, 5]. There is no satisfactory explanation for the thickness dependence of the toughness, although some models for the thickness effect have been proposed [6–9].

A recent study [10] on the fracture of polyester films indicated that the fracture toughness in the pure-plane-stress region increases with increasing thickness. The characterizing parameter for fracture toughness was that of the essential work of fracture originally proposed by Broberg [11].

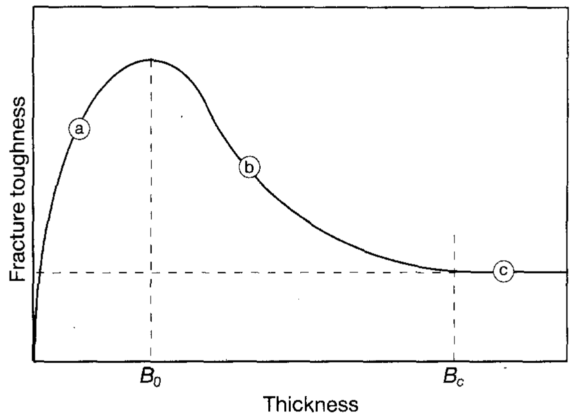

Figure 1 Toughness as a function of thickness: (a) pure plane stress, (b) plane stress/strain, and (c) pure plane strain.

The aim of this paper is to measure the essential work of fracture, w_e, for polycarbonate films of varying thicknesses and hence plot a graph of w_e versus thickness. In addition, several specimens of varying widths were also tested in order to establish whether the measured value of w_e is dependent on this dimension of the test piece.

2. Determination of the essential work of fracture

Broberg [11] proposed that the non-elastic region at the tip of a crack may be divided into two regions: an end region where the fracture process takes place, and an outer region where screening plastic deformation is necessary to accommodate the large strains in the end region (see Fig. 2). Following Broberg's suggestion, several investigators [12–15] have characterized ductile fracture in polymeric materials by partitioning the total work of fracture, W_f, into two parts: (i) work that is expended in the fracture-process zone, W_e, which is regarded as being essential for the fracture process in the formation of a neck and which subsequently initiates tearing of the neck; and (ii) work which is responsible for plastic deformation, W_p, but which is not essential for the fracture process. Hence, non-essential work is that work which is dissipated in the plastic zone outside the fracture-process zone. The total fracture work may therefore be written as:

$$W_f = W_e + W_p \quad (3)$$

where, W_e, is proportional to the ligament length, L, and W_p is proportional to L^2. Thus

$$W_f = LBw_e + \beta L^2 Bw_p \quad (4)$$

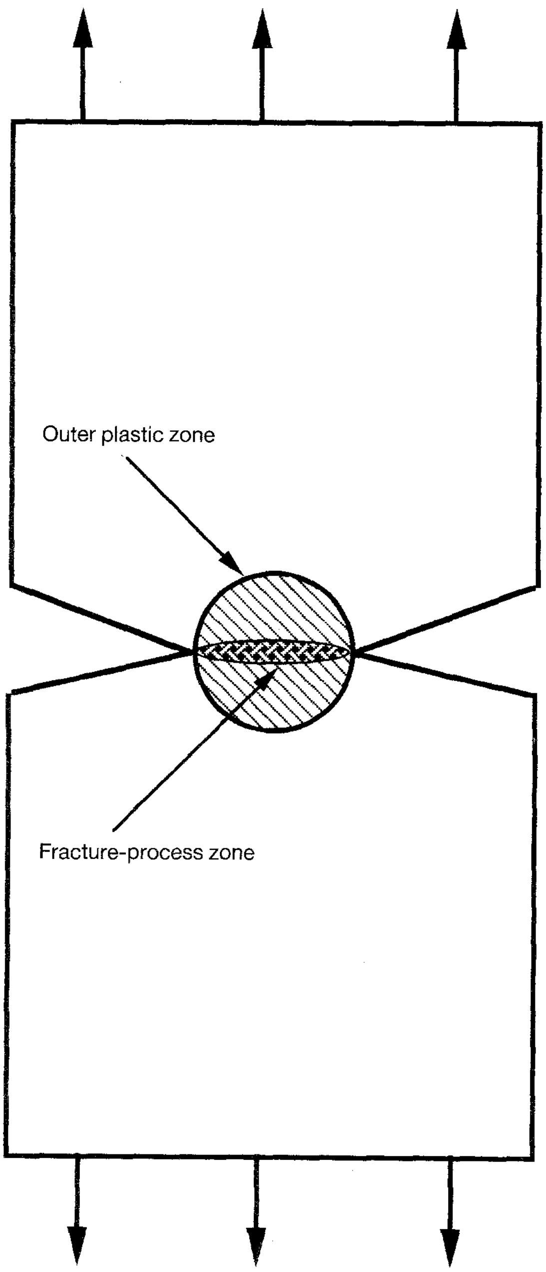

Figure 2 Crack-tip-deformation zone.

In Equation 4, w_e represents the work that is consumed per unit area in the fracture-process zone and is termed the specific essential work of fracture. This parameter is regarded as a material property for a given thickness. w_p is the work dissipated per unit volume of the material. β is a shape factor for the outer plastic zone; its value depends upon the geometry of the specimen and the crack. The term βw_p is not regarded as a material property, but is a measure of the plastic deformation around the crack tip. Rewriting Equation 4 gives

$$w_f = \frac{W_f}{LB} = w_e + \beta w_p L \quad (5)$$

where, w_f, is the specific total work of fracture (total work of fracture per unit ligament area). Equation 5 predicts a linear relationship between w_f and L (see Fig. 3) having a positive intercept at $L = 0$ to give w_e; and a slope that is proportional to w_p. For a given material w_p would be expected to increase with ductility or otherwise approach a value of zero with increasing degree of brittleness. It is noteworthy that Equation 5 assumes that the ligament length, L, controls the size of the plastic zone and that the volume of this zone is proportional to L^2B with the shape factor, β, being the proportionality constant. This proportionality may be affected in two ways.

1. First, if L is not small compared to the total width of the sample, then the size of the plastic zone can be disturbed by edge effects. To avoid these effects it is recommended that L should be kept below $W/3$.
2. Secondly, if the ligament length is larger than twice the radius of the plastic zone around the crack tip then the ligament area would not yield completely at failure.

Under these conditions L will not control the size of the plastic zone and in order to avoid this problem it is proposed [12] that L should be smaller than the minimum of the two following criteria:

$$L \leq W/3, \qquad L \leq 2r_p$$

where, r_p, is the radius of the plastic zone which is

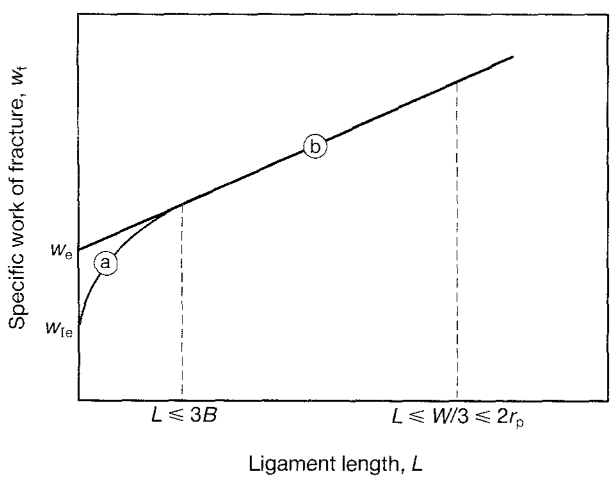

Figure 3 Schematic representation of the specific work of fracture versus ligament length: (a) plane stress/strain, and (b) pure plane stress.

given by LEFM (Linear elastic fracture mechanics) as $(1/2\pi)(K_c/\sigma_y)^2$ for a zone having a circular shape and $(\pi/8)(K_c/\sigma_y)^2$ for a linear plastic zone. A further factor to take into consideration is that the size of the test specimen must be chosen such that w_e, w_p and β are all independent of L. To achieve this, the state of pure plane stress must always exist in the specimen and this imposes a lower limit on the size of the ligament length which is governed by the sheet thickness, B. It has been shown [12, 13] that for ligament lengths smaller than $3B$ transition from pure-plane-stress fracture to a mixed-mode fracture (plane stress/plane strain) may be expected, giving rise to a non-linear relationship between w_f and L due to the increasing plastic-flow constraint with decreasing ligament length. It is therefore suggested that in order to meet the practical requirements, fracture specimens must satisfy the following size criterion:

$$3B \leq L \leq \min(W/3, 2r_p) \quad (6)$$

3. Experimental details

The material used for this investigation was polycarbonate. The material was supplied by Bayer in the form of films of nominal thicknesses 175, 250, 375 and 520 μm under the trade name Makrofol DE. Standard test specimens in the form of dumbells were cut from the films and pulled in an Instron testing machine at a constant crosshead speed of $1\ \text{mm}\,\text{min}^{-1}$. The load–displacement curves produced under these testing conditions showed a clear yield point (Fig. 4). From the maximum load on these diagrams and the original cross-sectional area of the test specimens, an average yield stress value of 57.10 MPa was determined.

Fracture tests were carried out using single-edge-notched-tension (SENT) specimens (see Fig. 5) of various ligament lengths, L. These specimens were cut from the polycarbonate films in such a manner that the fracture plane was always perpendicular to the machine direction. All the specimens were razor notched and tested on an Instron testing machine using pneumatic grips. All the tests were performed at room temperature using a constant crosshead speed of $1\ \text{mm}\,\text{min}^{-1}$.

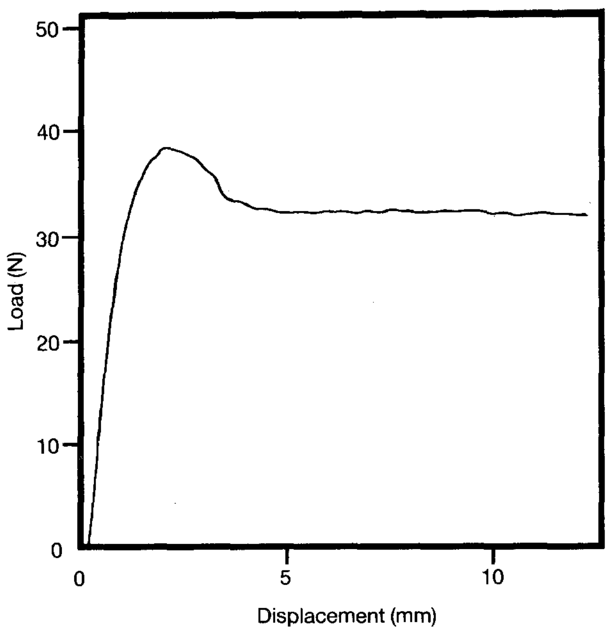

Figure 4 A typical tensile load–displacement diagram for a polycarbonate film.

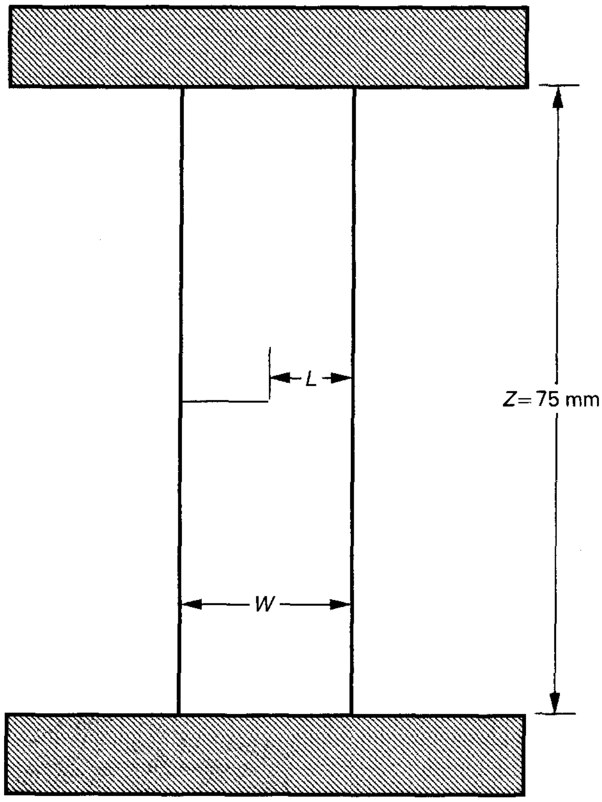

Figure 5 SENT geometry and the dimensions of the test specimens.

	B(μm)			
	175	250	375	520
W(mm)	25 15 35	25	25	25

4. Results and discussion

The razor-notching procedure produced a sharp crack which could be seen to open as the specimen was loaded. Further increases in load led to slight crack-tip blunting and the development of a necked-down crack-tip-deformation zone in the form of a line plastic zone (Dugdale plastic zone) at the tip of the crack (see Fig. 6). It is noteworthy that this necked-down crack-tip-deformation zone was always considerably wider than the film thickness. Seemingly, the manner in which test specimens fractured was dependent upon the ligament length. Two types of fracture behaviour were noted.

1. When the ligament length was short, the crack was seen to initiate after the ligament area completely yielded. The crack propagated slowly within the yielded zone until it eventually reached the back of the test specimen. At this point the test specimen was fractured.

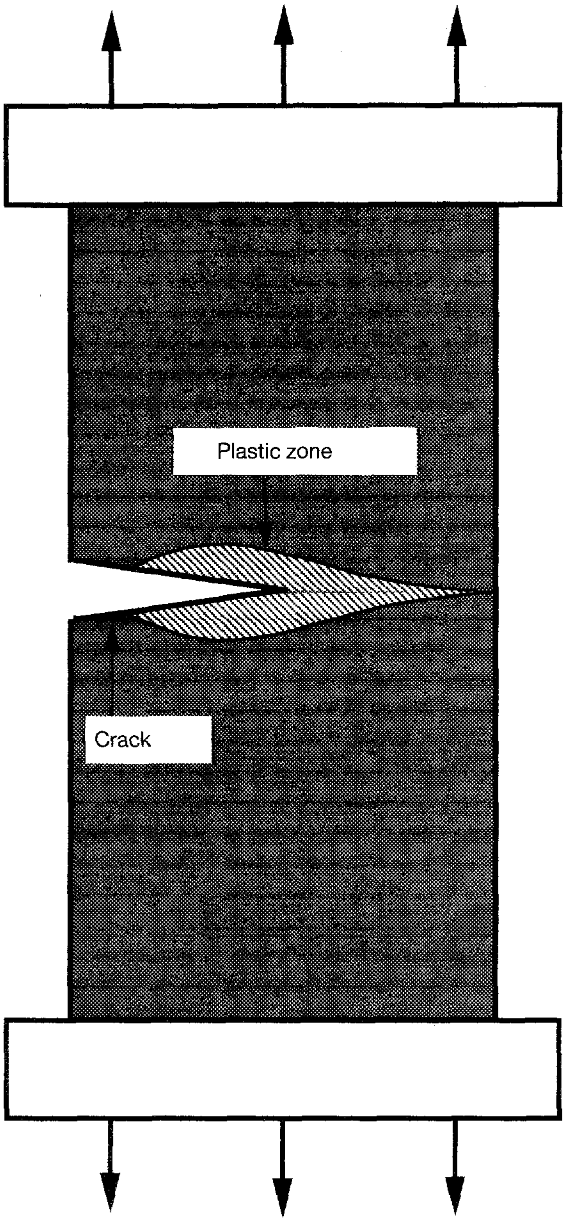

Figure 6 Schematic representation of the plastic zone in polycarbonate films.

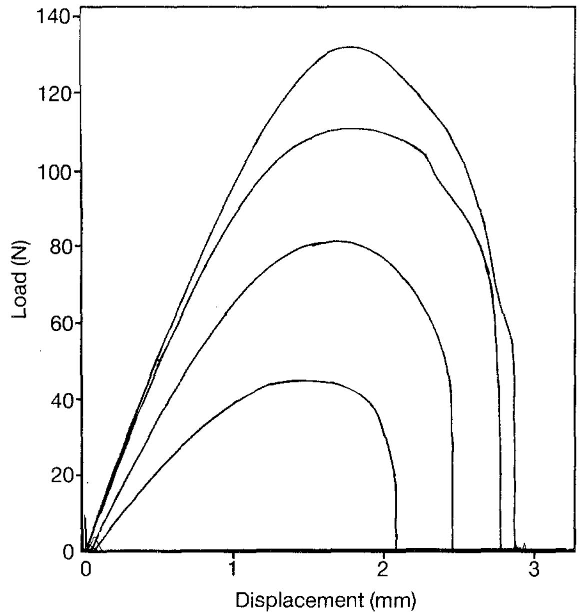

Figure 7 Typical load–displacement diagram produced by a SENT specimen with ligament-length values of 6.55, 11.92, 15.48 and 18.4 mm.

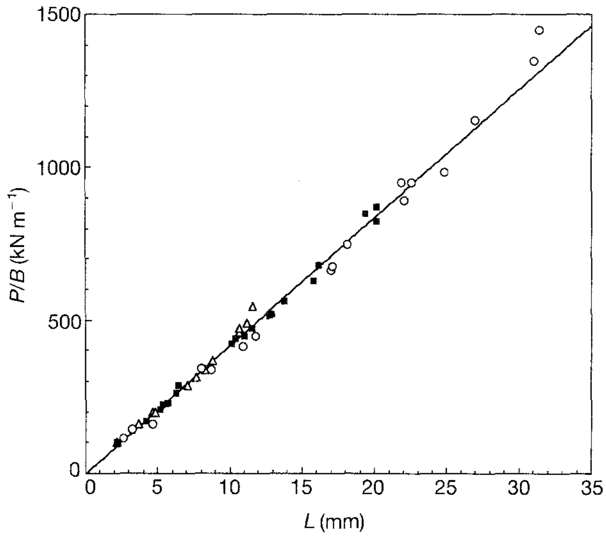

Figure 8 Maximum load per unit thickness for 175 μm thick polycarbonate specimens of varying widths: (△) $w = 15$ mm, (■) $w = 25$ mm, and (○) $w = 35$ mm.

2. When the ligament length was long, the crack was seen to initiate before the ligament area had completely yielded. In this case, the slow crack growth was accompanied by progressive development of a yielded zone ahead of the crack tip. The specimen eventually fractured when the crack grew across the full width of the test specimen.

Typical load–displacement curves produced using SENT test specimens are shown in Fig. 7. The maximum load in these diagrams signifies the load at which sufficient yielding in the ligament area had taken place, so that no higher load could be sustained by the material in the ligament area. Further propagation of the crack, therefore, occurred under a decreasing load by a process of ductile tearing under plane-stress conditions until the load reached zero. The maximum load in these diagrams is plotted against the ligament length in Fig. 8. Evidently, the relationship between P_{max} and L is linear and is almost independent of the sample width, W. According to Hill [16] the relationship between the maximum load and the ligament length for SENT specimens may be described as;

$$P_{max} = m\sigma_y LB \quad (7)$$

where m is the plastic-constraint factor whose value can be determined from the slope of the line. It is noteworthy, that the value of m obtained from the slope of the line in Fig. 8 is 0.72, which is considerably smaller than the value 1.12 given in Hill's analysis. This difference in the m-values may be explained by considering the nature and the shape of the plastic zone in these specimens. Firstly, Hill's analysis is based on a rigid plastic zone which is, of course quite different to the line plastic zone observed here. Secondly, the maximum load in Hill's analysis represents the load at which the ligament area has completely yielded, whereas the ligament area in most specimens tested here had not completely yielded when the maximum load was reached. Thirdly, the initial crack was seen to grow prior to the attainment of the maximum load, thus implying that the ligament length in the specimens at the maximum load was always smaller than the ligament length at the beginning of the test. The latter suggests that perhaps the ligament-length values used to plot Fig. 8 must have been those corresponding to P_{max} and not to the initial values. However, since these ligament-length values were not measured, the initial values were used in the maximum load analysis instead.

To determine the total specific work of fracture, w_f, areas under load–displacement diagrams were calculated and then plotted against the ligament length, L, as shown in Fig. 9. The data shown in Fig. 9 covers a wide range of specimen width and ligament-length values. Evidently, for 15 mm and 25 mm wide specimens, the relationship between w_f and L is linear for all values of L and shows no significant width effect. However, the data obtained using 35 mm wide specimens clearly shows non-linearity at large values of L, but at low values of L the data falls on the same line as for the two narrower specimens. It is interesting to note that, whilst the condition $L > 3B$ was comfortably met by all the specimens, the condition $L < \min(W/3 \text{ or } 2r_p)$ was not always adhered to. If the line in Fig. 9 is now extrapolated to zero ligament length, a specific essential work of fracture is obtained of 29 kJ m^{-2} for 175 μm thick film. Using this w_e-value, a tensile yield stress of 57.1 MPa and a Young's modulus of 2 GPa, then $2r_p$ for a line plastic zone is obtained as

$$2r_p = \frac{\pi}{8}\left(\frac{Ew_e}{\sigma_y^2}\right) = 6.8 \text{ mm} \quad (8)$$

Evidently, the data of Fig. 9 show no apparent deviation from linearity at the ligament-length value corresponding to 6.5 mm. This suggests that the ligament-length requirement of Equation 6 may indeed be too stringent when the test specimens are very thin and the plastic-zone shape is not circular.

Fig. 10 shows a plot of the maximum load per unit thickness versus ligament length for several film thicknesses. The observed behaviour is linear as in Fig. 8 and with the same slope giving an m-value of 0.72 as before. The effect of specimen thickness on w_e can be studied using the data presented in Figs 11–13. As before, w_f varies linearly with L with no apparent deviation from linearity even though L-values exceeded 6.8 mm. Extrapolating the lines in these figures to $L = 0$ gives the w_e-values listed in Table I, where it can be seen that the specific essential work of fracture, w_e, increases with increasing thickness. Also given in Table I are values of βw_p obtained from the slope of these lines. Evidently, for 175 μm

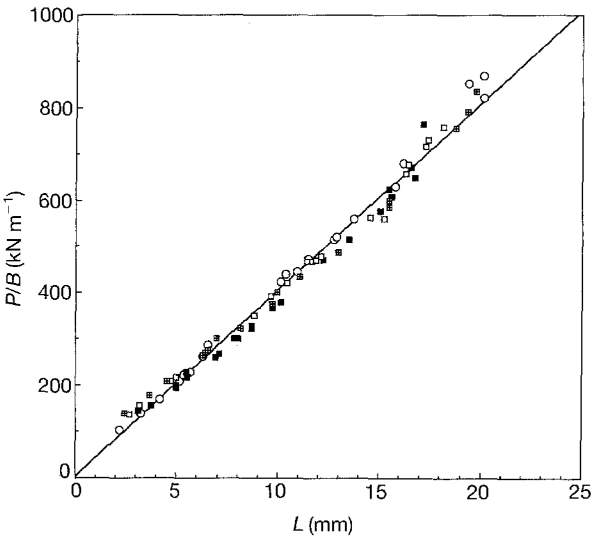

Figure 10 Maximum load per unit thickness for 25 mm wide SENT specimens with varying thicknesses: (○) $B = 175$ μm, (■) $B = 250$ μm, (□) $B = 375$ μm, and (▦) $B = 520$ μm.

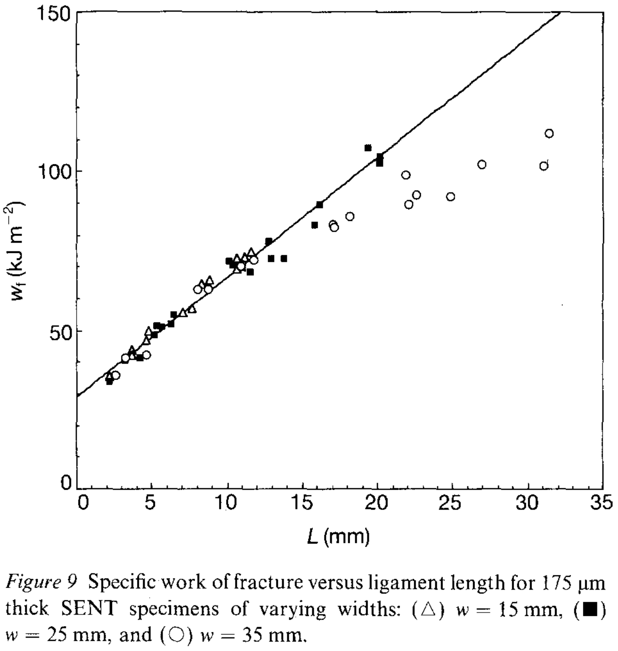

Figure 9 Specific work of fracture versus ligament length for 175 μm thick SENT specimens of varying widths: (△) $w = 15$ mm, (■) $w = 25$ mm, and (○) $w = 35$ mm.

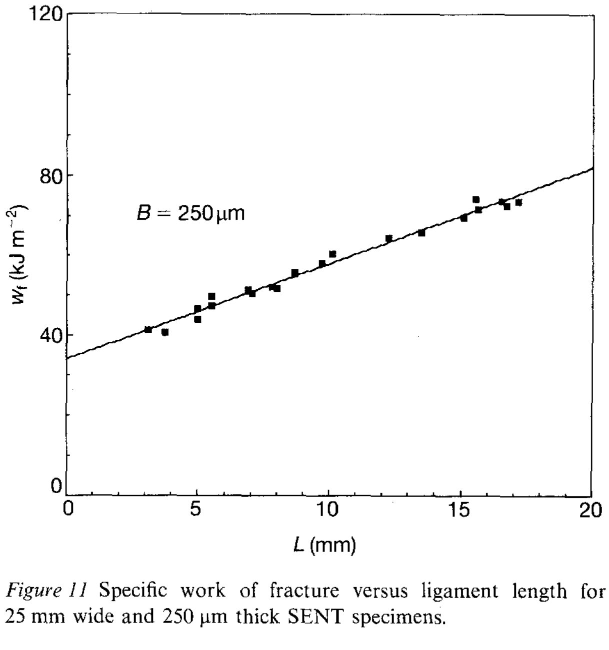

Figure 11 Specific work of fracture versus ligament length for 25 mm wide and 250 μm thick SENT specimens.

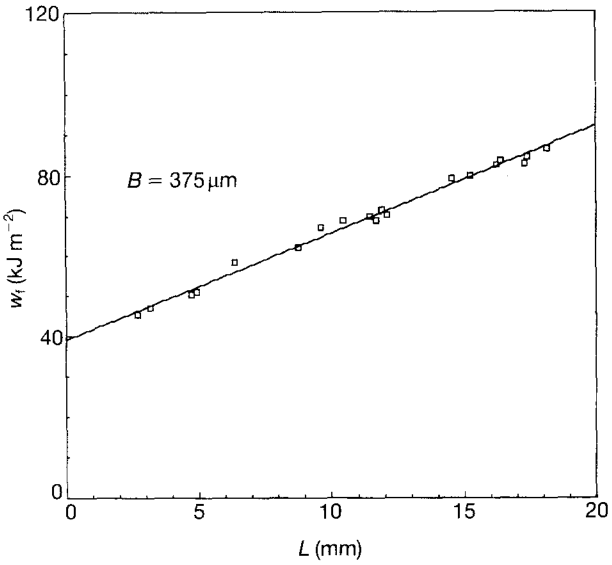

Figure 12 Specific work of fracture versus ligament length for 25 mm wide and 375 μm thick SENT specimens.

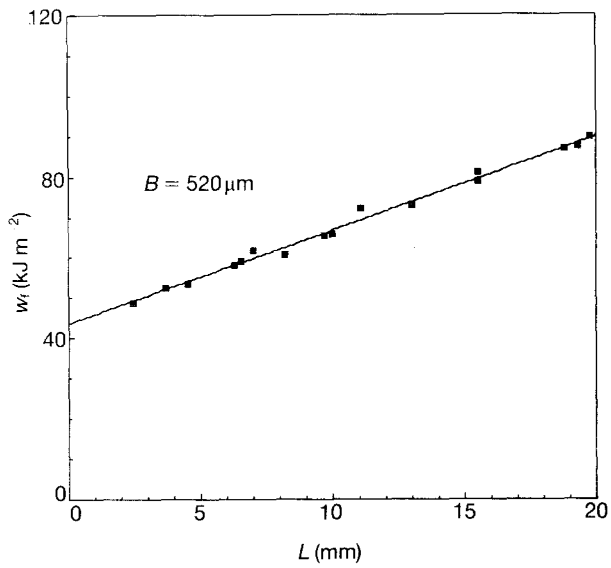

Figure 13 Specific work of fracture versus ligament length for 25 mm wide and 520 μm thick SENT specimens.

TABLE I Fracture data for several film thicknesses

Thickness, B(mm)	w_e(kJ m^{-2})	βw_e(MJ m^{-3})
175	29.00	4.12
250	34.13	2.40
375	39.18	2.65
520	43.70	2.35

thick film, the specific non-essential work of fracture, βw_p, is somewhat higher than the values obtained for other thicknesses. Seemingly, values of βw_p for 250, 375 and 520 μm thick films are similar, which indicates that the shape of the plastic zone surrounding the fracture-process zone is invariant with thickness. It must be pointed out that the shape of the plastic zone in 175 μm thick film did not seem different to that for

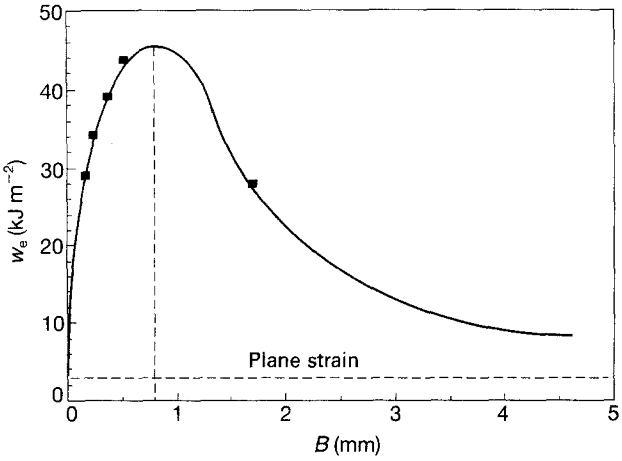

Figure 14 Specific essential work of fracture versus thickness.

thicker films, thus the higher βw_p-value for this film is not due to having a different plastic-zone shape.

5. Conclusion

A previous study [13] on a polycarbonate material 1.7 mm thick resulted in a w_e-value of 28 kJ m^{-2}. Also, by limiting the ligament length to less than three times the sample thickness, some fracture data for polycarbonate in the plane-strain/plane-stress transition region was ascertained. Extrapolation of this data to a zero ligament length gave a plane-strain specific essential work, w_{Ie}, value of 3 kJ m^{-2}. Fig. 14 shows a plot of w_e for polycarbonate versus thickness. Evidently, the variation in w_e with thickness closely resembles the behaviour described in Fig. 1. However, rewriting Equation 2 in terms of w_e gives

$$B_0 = \frac{1}{3\pi}\left(\frac{Ew_{Ie}}{\sigma_y^2}\right) \quad (9)$$

which, when values for E, σ_y and w_{Ie} are substituted, gives an optimum thickness, B_0, value of 0.2 mm, where w_e is expected to reach its highest level. According to Fig. 14, w_e reaches its highest level around 0.5 mm, which is almost 2.5 times the thickness value predicted by Equation 9. It must be noted that Equation 9 is based on a circular plastic zone; for a line plastic zone Equation 9 may be written as $(\pi/24)(Ew_{Ie}/\sigma_y^2)$ which gives a slightly higher value of 0.250 mm for B_0 but is still well below the value observed experimentally.

From the results presented in this paper it may be concluded that the specific essential work of fracture is a useful parameter for characterizing fracture toughness in thin films which fail under plane-stress conditions; its value is dependent on the thickness of the film, but is independent of the width of the test specimen.

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References

1. W. F. BROWN and J. E. STRAWLY, ASTM STP 410 (American Society for Testing and Materials, Philadelphia, 1966).
2. D. BROEK, "Elementary engineering fracture mechanics, 3rd Edn, (Martinus Nijhoff, Leiden 1982).
3. F. C. ALLEN, ASTM STP 486 (American Society for Testing and Materials, Philadelphia, 1971) 16.
4. V. WEISS and S. YUKAWA, ASTM STP 381 (American Society for Testing and Materials, Philadelphia, 1965) 1.
5. R. H. CHRISTENSEN and P. H. DENKE, ASD-TR-61-207 (1961).
6. J. I. BLUHM, *ASTM Proc.* **61** (1961) 1324.
7. G. C. SIH and R. J. HARTRANFT, *Int. J. Fract.* **9** (1973) 75.
8. D. P. ISHERWOOD and J. G. WILLIAMS, *Engng. Fract. Mech.* **2** (1970) 19.
9. W. E. ANDERSON, *Battelle Northwest Report. SA-2290* (1969).
10. S. HASHEMI, *J. Plas. Rub. Comp. Proc. Appl.* submitted.
11. K. B. BROBERG, *Int. J. Fract.* **4** (1968) 11.
12. Y. W. MAI and B. COTTERELL, *Int. J. Fract.* **32** (1986) 105.
13. C. A. PATON and S. HASHEMI. *J. Mater. Sci.* **27** (1992) 2279.
14. A. S. SALEEMI and J. A. NAIRN, *Polym. Engng. Sci.* **30** (1990) 211.
15. Y. W. MAI and P. POWELL, *J. Polym. Sci. B* **29** (1991) 785.
16. R. H. HILL, *J. Mech. Phys. Solids.* **4** (1952) 19.

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